THIRD EDITION

PERMANENT MAGNET MOTOR TECHNOLOGY

Design and Applications

JACEK F. GIERAS

CRC Press
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Preface to the Third Edition

The importance of permanent magnet (PM) motor technology and its impact on electromechanical drives has significantly increased since publication of the first edition of this book in 1996 and the second edition in 2002. There was 165% PM brushless motor market growth between 2005 and 2008 in comparison with 29% overall motion control market growth in the same period of time.

It is expected that the development of electric machines, mostly PM machines and associated power electronics in the next few years will be stimulated by large scale applications such as (a) computer hardware, (b) residential and public applications, (c) land, sea and air transportation and (d) renewable energy generation [124]. The development of PM machines is, however, not limited to these four major areas of application since PM machines are vital apparatus in all sectors of modern society, such as industry, services, trade, infrastructure, healthcare, defense, and domestic life. For example, worldwide demand on PM vibration motors for mobile phones increases at least 8% each year and global shipment of hard disk drives (HDD) with PM brushless motors increases about 24% each year.

In the last two decades new topologies of high torque density PM motors, high speed PM motors, integrated PM motor drives, and special PM motors have gained maturity. The largest PM brushless motor in the world rated at 36.5 MW, 127 rpm was built in 2006 by DRS Technologies, Parsippany, NJ, U.S.A.

In comparison with the 2002 edition, the 3rd edition has been thoroughly revised and updated, new chapters on high speed motors and micromotors have been written and more numerical examples and illustrative material have been added.

Any critical remarks, corrections, and suggestions for improvements are most welcome and may be sent to the author at jgieras@ieee.org.

Prof. Jacek F. Gieras, IEEE Fellow

1

Introduction

1.1 Permanent magnet versus electromagnetic excitation

The use of permanent magnets (PMs) in construction of electrical machines brings the following benefits:

- no electrical energy is absorbed by the field excitation system and thus there are no excitation losses which means substantial increase in efficiency,
- higher power density and/or torque density than when using electromagnetic excitation,
- better dynamic performance than motors with electromagnetic excitation (higher magnetic flux density in the air gap),
- simplification of construction and maintenance,
- reduction of prices for some types of machines.

The first PM excitation systems were applied to electrical machines as early as the 19th century, e.g., J. Henry (1831), H. Pixii (1832), W. Ritchie (1833), F. Watkins (1835), T. Davenport (1837), M.H. Jacobi (1839) [35]. Of course, the use of very poor quality hard magnetic materials (steel or tungsten steel) soon discouraged their use in favor of electromagnetic excitation systems. The invention of Alnico in 1932 revived PM excitation systems; however, its application was limited to small and fractional horsepower d.c. commutator machines. At the present time most PM *d.c. commutator motors* with slotted rotors use ferrite magnets. Cost effective and simple d.c. commutator motors with *barium* or *strontium ferrite PMs* mounted on the stator will still be used in the forseeable future in road vehicles, toys, and household equipment.

Cage induction motors have been the most popular electric motors in the 20th century. Recently, owing to the dynamic progress made in the field of power electronics and control technology, their application to electrical drives has increased. Their rated ouput power ranges from 70 W to 500 kW, with 75% of them running at 1500 rpm. The main advantages of cage induction motors are their simple construction, simple maintenance, no commutator or slip rings, low price and moderate reliability. The disadvantages are their small air gap, the possibility of cracking the rotor bars due to hot spots at plugging and reversal, and lower efficiency and power factor than synchronous motors.

The use of *PM brushless motors* has become a more attractive option than induction motors. *Rare earth PMs* can not only improve the motor's steady-state performance but also the power density (output power-to-mass ratio), dynamic performance, and quality. The prices of rare earth magnets are also dropping, which is making these motors more popular. The improvements made in the field of semiconductor drives have meant that the control of brushless motors has become easier and cost effective, with the possibility of operating the motor over a large range of speeds while still maintaining good efficiency.

Servo motor technology has changed in recent years from conventional d.c. or two-phase a.c. motor drives to new maintenance-free brushless three-phase vector-controlled a.c. drives for all motor applications where quick response, light weight and large continuous and peak torques are required.

A PM brushless motor has the magnets mounted on the rotor and the armature winding mounted on the stator. Thus, the armature current is not transmitted through a commutator or slip rings and brushes. These are the major parts which require maintenance. A standard maintenance routine in 90% of motors relates to the sliding contact. In a d.c. commutator motor the power losses occur mainly in the rotor which limits the heat transfer and consequently the armature winding current density. In PM brushless motors the power losses are practically all in the stator where heat can be easily transferred through the ribbed frame or, in larger machines, water cooling systems can be used [9, 24, 223]. Considerable improvements in dynamics of brushless PM motor drives can be achieved since the rotor has a lower inertia and there is a high air gap magnetic flux density and no-speed dependent current limitation.

The *PM brushless motor electromechanical drive* has become a more viable option than its induction or reluctance counterpart in motor sizes up to 10 – 15 kW. There have also been successful attempts to build PM brushless motors rated above 1 MW (Germany and U.S.A.) [9, 23, 24, 124, 223, 276]. The high performance rare-earth magnets have successfully replaced ferrite and Alnico magnets in all applications where high power density, improved dynamic performance or higher efficiency are of prime interest. Typical examples where these points are key selection criteria are stepping motors for computer peripheral applications and servo motors for machine tools or robotics.

1.2 Permanent magnet motor drives

In general, all electromechanical drives can be divided into constant-speed drives, servo drives and variable-speed drives.

A *constant-speed drive* usually employs a synchronous motor alone which can keep the speed constant without an electronic converter and feedback or any other motor when there is less restriction on the speed variation tolerance.

A *servo system* is a system consisting of several devices which continuously monitor actual information (speed, position), compare these values to desired outcome and make necessary corrections to minimize the difference. A *servo motor drive* is a drive with a speed or position feedback for precise control where the response time and the accuracy with which the motor follows the speed and position commands are extremely important.

In a *variable-speed drive* (VSD) the accuracy and the response time with which the motor follows the speed command are not important, but the main requirement is to change the speed over a wide range.

In all electromechanical drives where the speed and position are controlled, a *solid state converter* interfaces the power supply and the motor. There are three types of PM motor electromechanical drives:

- d.c. commutator motor drives
- brushless motor drives (d.c. and a.c. synchronous)
- stepping motor drives

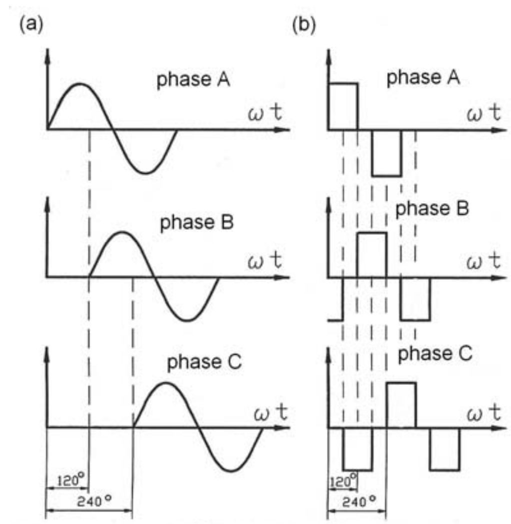

Fig. 1.1. Basic armature waveforms for three phase PM brushless motors: (a) sinusoidally excited, (b) square wave.

Brushless motor drives fall into the two principal classes of *sinusoidally excited* and *square wave* (trapezoidally excited) motors. Sinusoidally excited motors are fed with three-phase sinusoidal waveforms (Fig. 1.1a) and operate on the principle of a rotating magnetic field. They are simply called *sinewave motors* or *PM synchronous motors*. All phase windings conduct current at a time. Square wave motors are also fed with three-phase waveforms shifted by 120^0 one from another, but these waveshapes are rectangular or trapezoidal (Fig. 1.1b). Such a shape is produced when the armature current (MMF) is precisely synchronized with the rotor instantaneous position and frequency (speed). The most direct and popular method of providing the required rotor position information is to use an absolute angular position sensor mounted on the rotor shaft. Only two phase windings out of three conduct current simultaneously. Such a control scheme or *electronic commutation* is functionally equivalent to the mechanical commutation in d.c. motors. This explains why motors with square wave excitation are called *d.c. brushless motors*. An alternative name used in power electronics and motion control is *self-controlled synchronization* [166].

Although stepping motor electromechanical drives are a kind of synchronous motor drive, they are separately discussed due to their different control strategies and power electronic circuits.

1.2.1 d.c. commutator motor drives

The *d.c. commutator motor* or *d.c brush motor* is still a simple and low cost solution to variable-speed drive systems when requirements such as freedom from maintenance, operation under adverse conditions or the need to operate groups of machines in synchronism are not supreme [167]. Owing to the action of the mechanical commutator, control of a d.c. motor drive is comparatively simple and the same basic control system can satisfy the requirements of most applications. For these reasons the d.c. electromechanical drive very often turns into the cheapest alternative, in spite of the cost of the d.c. commutator motor. In many industrial applications such as agitators, extruders, kneading machines, printing machines, coating machines, some types of textile machinery, fans, blowers, simple machine tools, etc., the motor is required only to start smoothly and drive the machinery in one direction without braking or reverse running. Such a drive operates only in the first quadrant of the speed-torque characteristic and requires only one *controlled converter* (in its rectifier mode) as shown in Fig. 1.2a. At the expense of increased torque ripple and supply harmonics, a half-controlled rather than fully-controlled bridge may be used up to about 100 kW. If the motor is required to drive in both forward and reverse directions, and apply regenerative braking, a single fully controlled converter can still be used but with the possibility of reversing the armature current (Fig. 1.2a).

Electromechanical drives such as for rolling mills, cranes and mine winders are subject to rapid changes in speed or in load. Similarly, in those textile, paper or plastics machines where rapid control of tension is needed, frequent small speed adjustments may request rapid torque reversals. In these cases a *four-quadrant dual converter* comprising two semiconductor bridges in antiparallel, as in Fig. 1.2b, can be used [167]. One bridge conducts when ar-

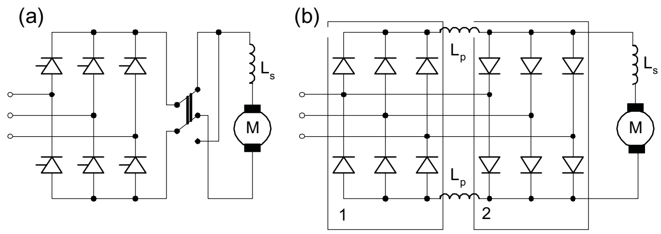

Fig. 1.2. d.c. commutator motor drives: (a) one quadrant, fully-controlled, single-converter drive, (b) four-quadrant, fully-controlled, dual converter drive; L_p are reactors limiting the currents circulating between the rectifying and inverting bridges, L_a is the armature circuit inductance.

mature current is required to be positive, and the second bridge when it is required to be negative.

Natural or *line commutation* between successively conducting power semiconductor switches takes place when the instantaneous values of consecutive phase voltages are equal and cross each other. The phase with a decreasing voltage is suppressed, while that with an increasing voltage takes over the conduction. Natural commutation may not be possible in such situations when the armature inductance is very small, e.g., control of small d.c. motors. The normal phase control would cause unacceptable torque ripple without substantial smoothing which, in turn, would impair the response of the motor [167]. Power transistors, GTO thyristors or IGBTs can be turned off by appropriate gate control signals, but conventional thyristors need to be reverse biased briefly for successful turn off. This can be accomplished by means of a forced-commutation circuit, usually consisting of capacitors, inductors and, in some designs, auxiliary thyristors. *Forced commutation* is used mostly for the frequency control of a.c. motors by variable-frequency inverters and chopper control of d.c. motors.

Fig. 1.3a shows the main components of a *one-directional chopper* circuit for controlling a PM or separately excited d.c. motor. Thyristors must be accompanied by some form of turn-off circuits or, alternatively, be replaced by GTO thyristors or IGBTs. When the mean value of the chopper output voltage is reduced below the armature EMF, the direction of the armature current cannot reverse unless T2 and D2 are added. The thyristor T2 is fired after T1 turns off, and vice versa. Now, the reversed armature current flows via T2 and increases when T1 is off. When T1 is fired, the armature current flows via D2 back to the supply. In this way regenerative braking can be achieved [167].

Full *four-quadrant operation* can be obtained with a bridge version of the chopper shown in Fig. 1.3b. Transistors or GTO thyristors allow the chopper to operate at the higher switching frequencies needed for low-inductance motors. By varying the on and off times of appropriate pairs of solid switches,

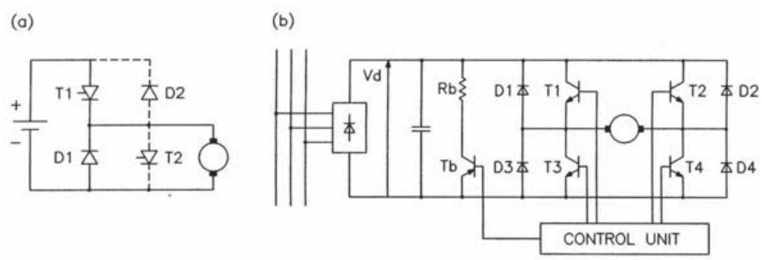

Fig. 1.3. Chopper controlled d.c. motor drives: (a) one-directional with added regenerative braking leg, (b) four-quadrant chopper controller [167].

the mean armature voltage, and hence speed, can be controlled in either direction. Typical applications are machine tools, generally with one motor and chopper for each axis, all fed from a common d.c. supply [167].

1.2.2 Synchronous motor drives

In a two-stage conversion a d.c. intermediate link is inserted between the line and motor-side converter (Fig. 1.4). For *low power PM synchronous motors* (in the range of kWs) a simple diode bridge rectifier as a line-side converter (Fig 1.4a) is used. The most widely used semiconductor switch at lower power permitting electronic turn-off is power transistor or IGBT.

Motor-side converters (inverters) have load commutation if the load, e.g., PM synchronous motor can provide the necessary reactive power for the solid state converter. Fig. 1.4b shows a basic power circuit of a *load-commutated current source thyristor converter*. The intermediate circuit energy is stored in the inductor. The inverter is a simple three-phase thyristor bridge. Load commutation is ensured by overexcitation of the synchronous motor so that it operates at a leading power factor (leading angle is approximately 30^0) [132]. This causes a decrease in the output power. The elimination of forced commutation means fewer components, simpler architecture, and consequently lower converter volume, mass, and losses. A four-quadrant operation is possible without any additional power circuitry. The motor phase EMFs required for load commutation of the inverter are not available at standstill and at very low speeds (less than 10% of the full speed). Under these conditions, the current commutation is provided by the line converter going into an inverter mode and forcing the d.c. link current to become zero, thus providing turn-off of thyristors in the load inverter [222].

The maximum output frequency of a load-commutated *current source inverter* (CSI) is limited by the time of commutation, which in turn is determined by the load. A CSI is suitable for loads of low impedance.

A *voltage source inverter* (VSI) is suitable for loads of high impedance. In a VSI the energy of the intermediate circuit is stored in the capacitor. A PWM VSI with GTOs and antiparallel diodes (Fig. 1.4c) allows a synchronous motor

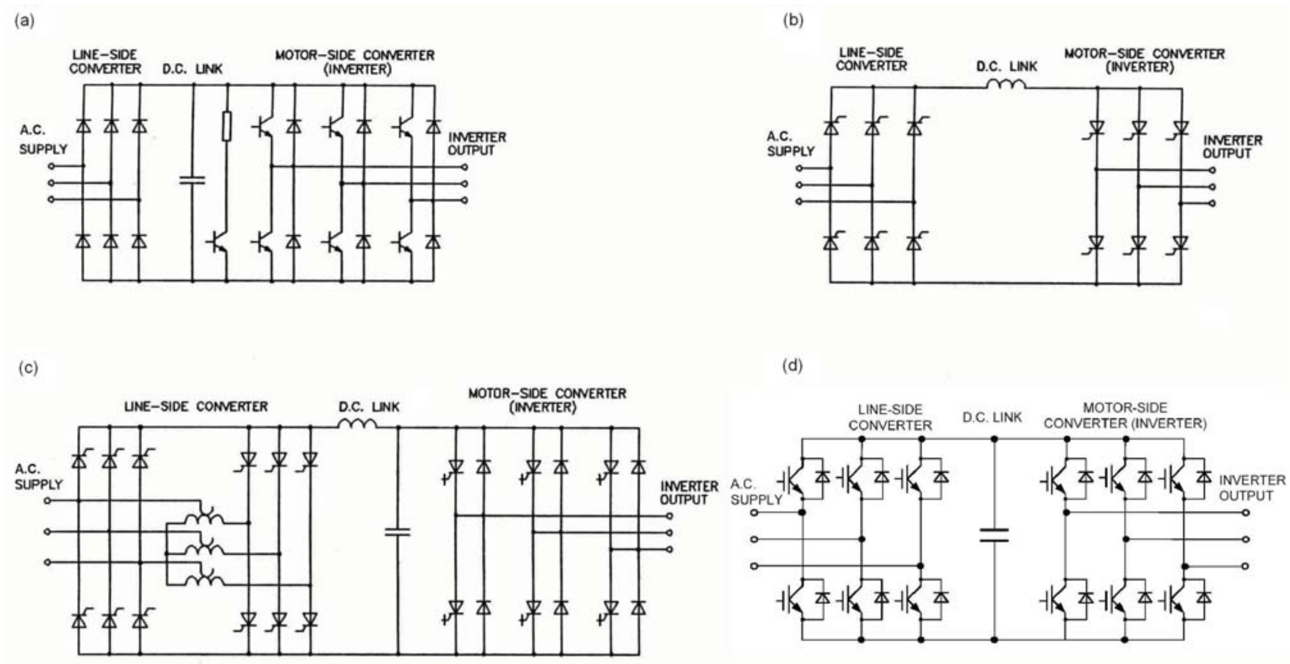

Fig. 1.4. Basic power circuits of d.c. link converters for synchronous motors with: (a) PWM transistor inverter, (b) load-commutated thyristor CSI, (c) forced-commutated GTO VSI (four-quadrant operation), (d) IGBT VSI.

to operate with unity power factor. Synchronous motors with high subtransient inductance can then be used. Four-quadrant operation is possible with a suitable power regeneration line-side converter. Replacement of thyristors by GTOs or IGBTs eliminates inverter commutation circuits and increases the pulse frequency.

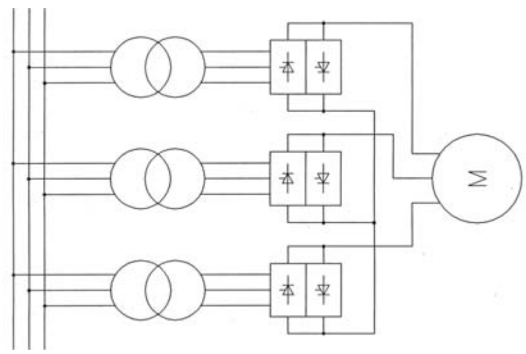

Fig. 1.5. Cycloconverter synchronous motor drive.

The maximum output frequency of the load-commutated CSI is limited to about 400 Hz even if fast thyristors are used. A higher output frequency can be achieved using an IGBT VSI with antiparallel diodes. Fig. 1.4d illustrates a typical PM brushless motor drive circuit with a three-phase PWM IGBT inverter. In the brushless d.c. mode, only two of the three phase windings are excited by properly switching the IGBTs of the inverter, resulting in ideal motor current waveforms of rectangular shape. There are six combinations of the stator winding excitation over a fundamental cycle with each combination lasting for a phase period of 60^o. The corresponding two active solid state switches in each period may perform PWM to regulate the motor current. To reduce current ripple, it is often useful to have one solid state switch doing PWM while keeping the other switch conducting.

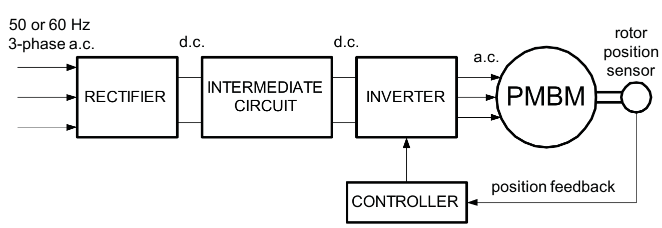

Fig. 1.6. d.c. brushless motor drive.

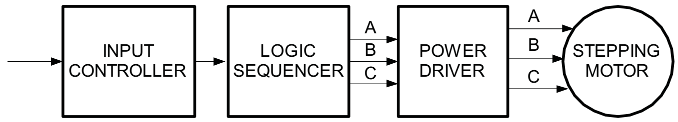

Fig. 1.7. Stepping motor drive.

A *cycloconverter* is a single stage (a.c. to a.c.) line-commutated frequency converter (Fig. 1.5). Four-quadrant operation is permitted as the power can flow in both directions between the line and the load. A cycloconverter covers narrow output frequency range, from zero to about 50% of the input frequency. Therefore, cycloconverters are usually used to supply gearless electromechanical drives with large power, low speed synchronous motors. For example, synchronous motors for ships propulsion are fed from diesel alternators mostly via cycloconverters [278]. A cycloconverter has an advantage of low torque harmonics of relatively high frequency. Drawbacks include large number of solid state switches, complex control, and poor power factor. Forced commuation can be employed to improve the power factor.

1.2.3 PM d.c. brushless motor drives

In PM d.c. brushless motors, square current waveforms are in synchronism with the rotor position angle. The basic elements of a *d.c. brushless motor drive* are: PM motor, output stage (inverter), line-side converter, shaft position sensor (encoder, resolver, Hall elements,), gate signal generator, current detector, and controller, e.g., microprocessor or computer with DSP board. A simplified block diagram is shown in Fig. 1.6.

1.2.4 Stepping motor drives

A typical *stepping motor drive* (Fig. 1.7) consists of an input controller, logic sequencer and driver. The input controller is a logic circuit that produces the required train of pulses. It can be a microprocessor or microcomputer which generates a pulse train to turn the rotor, speed up and slow down. The logic sequencer is a logic circuit that responds to step-command pulses and controls the excitation of windings sequentially [174]. Output signals of a logic sequencer are transmitted to the input terminals of a power drive which turns on and turns off the stepping motor windings. The stepping motor converts electric pulses into discrete angular displacements. The fundamental difference between a stepping motor drive and a *switched reluctance motor* (SRM) drive is that the first one operates in open loop control, without rotor position feedback.

1.3 Toward increasing the motor efficiency

Unforeseen consequences can result from problems the contemporary world currently faces, i.e.,

- fears of depletion and expected scarcities of major non-renewable energy resources over the next several decades
- increase in energy consumption
- pollution of our planet

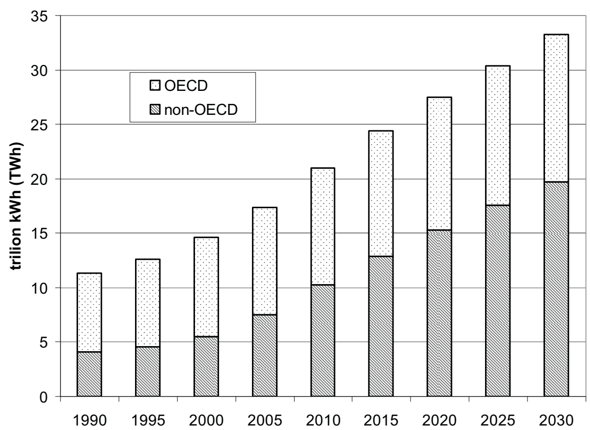

Fig. 1.8. World net electric power generation, 1990–2030 (history: 1990–2005, projections: 2005–2030) [158].

The world consumption of petroleum is about 84 million barrels per day (1 barrel = 159 l) or about 31 billion barells (about 5×10^{12} l) per year. If current laws and policies remain unchanged throughout the projection period, world marketed energy consumption is projected to grow by 50 percent over the 2005 to 2030 period [158].

About 30% of primary energy is used for generation of electricity. World net electricity generation will increase from 17,300 TWh (17.3 trillion kWh) in 2005 to 24,400 TWh in 2015 and 33,300 TWh in 2030 (Fig. 1.8). Total non-OECD [1] electricity generation increases by an average of 4.0 % per year, as compared with a projected average annual growth rate in OECD electricity generation of 1.3 % from 2005 to 2030 [158].

The mix of energy sources in the world is illustrated in Fig. 1.9. The 3.1 % projected annual growth rate for coal-fired electricity generation worldwide is

[1] The Organisation for Economic Co-operation and Development (OECD) is an international organisation of thirty countries that accept the principles of representative democracy and free-market economy.

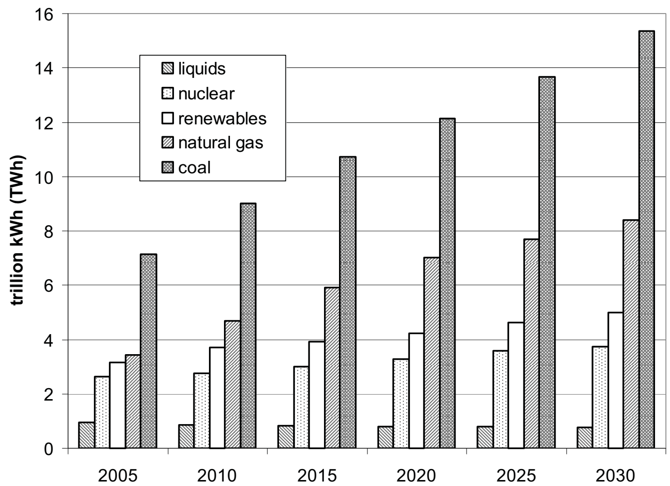

Fig. 1.9. World energy generation by fuel, 2005–2030 [158].

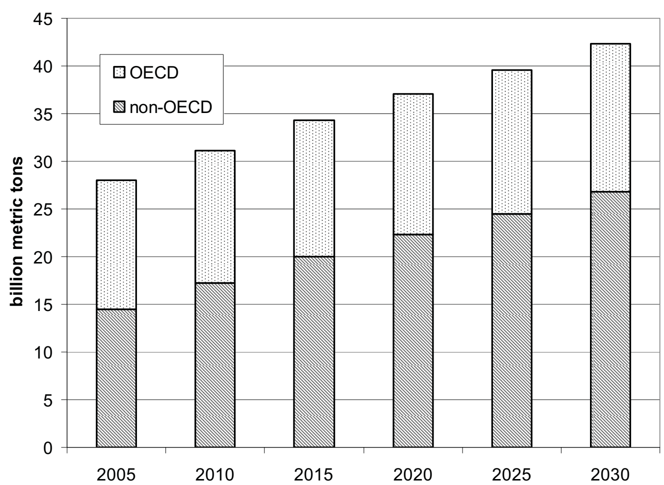

Fig. 1.10. World energy–related carbon dioxide emissions, 2005–2030 [158].

exceeded only by the 3.7-percent growth rate projected for natural-gas-fired generation [158].

The industrial sector, in developed countries, uses more than 30% of the electrical energy. More than 65% of this electrical energy is consumed by electric motor drives. The number of installed electrical machines can be estimated on the basis of their world production.

The increasing electrical energy demand causes tremendous concern for environmental pollution (Fig. 1.10). The power plants using fossil and nuclear fuel and road vehicles with combustion engines are main contributors to *air pollution, acid rain,* and the *greenhouse effect*. There is no doubt that electric propulsion and energy savings can improve these side effects considerably. For example, the population of Japan is about 50% of that of the U.S. However, carbon emmission is four times less (400 million metric ton in Japan versus over 1550 million metric ton in the U.S. in 2005). Mass public transport in Japan based on modern electrical commuter and long distance trains network plays an important part in reduction of carbon emission. It has been estimated that in developed industrialized countries, roughly 10% of electrical energy can be saved by using more efficient control strategies for electromechanical drives. This means that *electrical machines have an enormous influence on the reduction of energy consumption*. Electrical energy consumption can be saved in one of the following ways [212]:

- good housekeeping
- use of variable-speed drives
- construction of electric motors with better efficiency

Good housekeeping measures are inexpensive, quick and easy to implement. The simplest way to save energy costs is to switch idling motors off. Motors can be switched off manually or automatically. Devices exist that use either the input current to the motor or limit switches to detect an idling motor. When larger motors are being switched off and on, the high starting current drawn by the motor could cause supply interference and mechanical problems with couplings, gearboxes, belts, etc., which deteriorate from repeated starting. These problems can be avoided by using electronic *solid state converters*.

Fan and pump drives employ over 50% of motors used in industry. Most fans and pumps use some form of flow control in an attempt to match supply with demand. Traditionally, mechanical means have been used to restrict the flow, such as a damper on a fan or a throttle valve on a pump. Such methods waste energy by increasing the resistance to flow and by running the fan or pump away from its most efficient point. A much better method is to use a VSD to alter the speed of the motor. For centrifugal fans and pumps the power input is proportional to the cube of the speed, while the flow is proportional to the speed. Hence, a reduction to 80% of maximum speed (flow) will give a potential reduction in power consumption of 50% [212].

The application of PMs to electrical machines improves their efficiency by eliminating the excitation losses. The air gap magnetic flux density increases, which means greater output power for the same main dimensions.

A 3% increase in motor efficiency can save 2% of energy used [212]. Most energy is consumed by three-phase induction motors rated at below 10 kW. Consider a small three-phase, four-pole, 1.5-kW, 50-Hz cage induction motor. The full load efficiency of such a motor is usually 78%. By replacing this motor with a rare-earth PM brushless motor the efficiency can be increased to 89%. This means that the three-phase PM brushless motor draws from the mains only 1685 W instead of 1923 W drawn by the three phase cage induction motor. The power saving is 238 W per motor. If in a country, say, one million such motors are installed, the reduction in power consumption will be 238 MW, or one quite large turboalternator can be disconnected from the power system. It also means a reduction in CO_2 and NO_x emitted into the atmosphere if the energy is generated by thermal power plants.

1.4 Classification of permanent magnet electric motors

In general, rotary PM motors for continuous operation are classified into:

- d.c. brush commutator motors
- d.c. brushless motors
- a.c. synchronous motors

The construction of a PM d.c. commutator motor is similar to a d.c. motor with the electromagnetic excitation system replaced by PMs. PM d.c. brushless and a.c. synchronous motor designs are practically the same: with a polyphase stator and PMs located on the rotor. The only difference is in the control and shape of the excitation voltage: an a.c. synchronous motor is fed with more or less sinusoidal waveforms which in turn produce a rotating magnetic field. In PM d.c. brushless motors the armature current has a shape of a square (trapezoidal) waveform, only two phase windings (for Y connection) conduct the current at the same time, and the switching pattern is synchronized with the rotor angular position (electronic commutation).

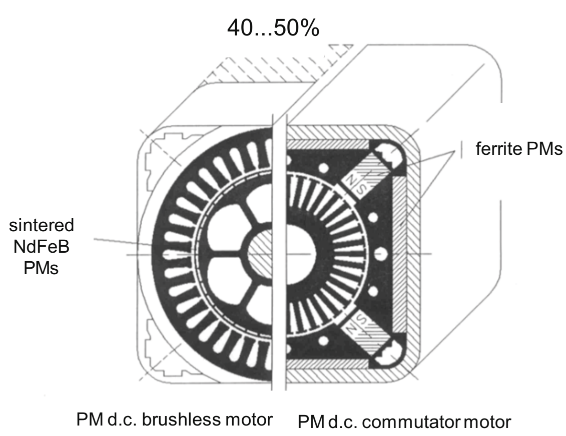

Fig. 1.11. Comparison of PM brushless and PM d.c. commutator motors.

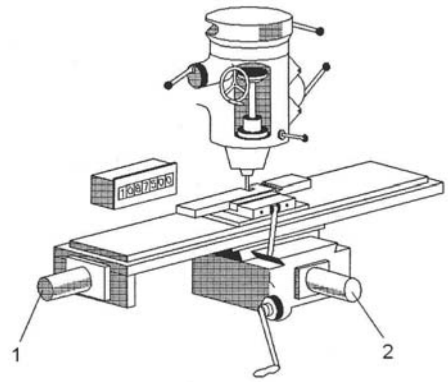

Fig. 1.12. Machine tool for milling grooves across a steel bar stock: 1 — PM brushless servo motor for indexing the stock, 2 — stepping motor for controlling the mill stroke. Courtesy of *Parker Hannifin Corporation*, Rohnert Park, CA, U.S.A.

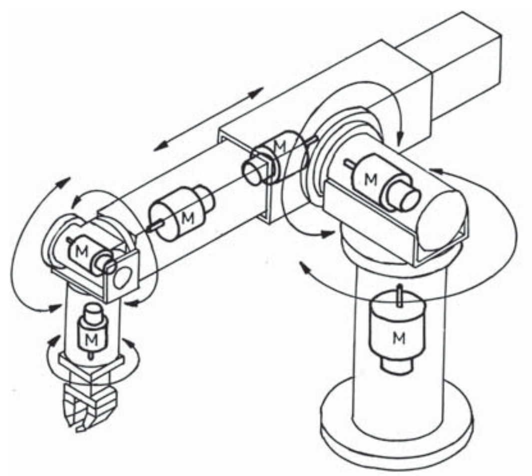

Fig. 1.13. An industrial robot (M — electric motor).

The armature current of synchronous and d.c. brushless motors is not transmitted through brushes, which are subject to wear and require maintenance. Another advantage of the brushless motor is the fact that the power losses occur in the stator, where heat transfer conditions are good. Consequently the power density can be increased as compared with a d.c. commutator motor. In addition, considerable improvements in dynamics can be achieved because the air gap magnetic flux density is high, the rotor has a lower inertia and there are no speed-dependent current limitations. Thus, the volume of a brushless PM motor can be reduced by 40 to 50% while still keeping the same rating as that of a PM commutator motor [75] (Fig. 1.11).

The following constructions of PM d.c. comutator motors have been developed:

- motors with conventional slotted rotors
- motors with slotless (surface-wound) rotors
- motors with moving coil rotors:
  (a) outside field type
  – cylindrical
  – wound disk rotor
  – printed circuit disk rotor
  (b) inside field type with cylindrical rotor
  – honeycomb armature winding
  – rhombic armature winding
  – bell armature winding
  – ball armature winding

The PM a.c. synchronous and d.c. brushless motors (moving magnet rotor) are designed as:

- motors with conventional slotted stators,
- motors with slotless (surface-wound) stators,
- cylindrical type:
  – surface magnet rotor (uniform thickness PMs, bread loaf PMs)
  – inset magnet rotor
  – interior magnet rotor (single layer PMs, double layer PMs)
  – rotor with buried magnets symmetrically distributed
  – rotor with buried magnets asymmetrically distributed
- disk type:
  (a) single-sided
  (b) double-sided
  – with internal rotor
  – with internal stator (armature)

The stator (armature) winding of PM brushless motors can be made of coils distributed in slots, concentrated non-overlapping coils or slotless coils.

1.5 Trends in permanent magnet motors and drives industry

The electromechanical drives market analysis shows that the d.c. commutator motor drive sales increase only slightly each year while the demand for a.c. motor drives increases substantially [288]. A similar tendency is seen in the PM d.c. commutator motor drives and PM brushless motor drives.

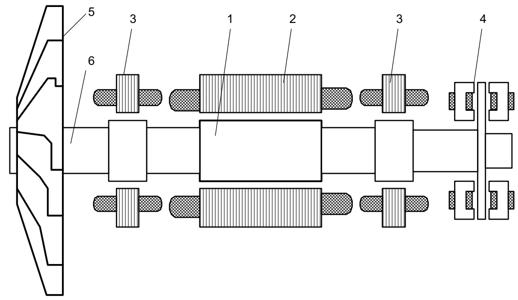

Fig. 1.14. Centrifugal chiller compressor with high speed PM brushless motor and magnetic bearings: 1 — rotor with surface PMs and nonmagnetic can, 2 — stator, 3 — radial magnetic bearing, 4 — axial magnetic bearing, 5 — impeller, 6 — shaft.

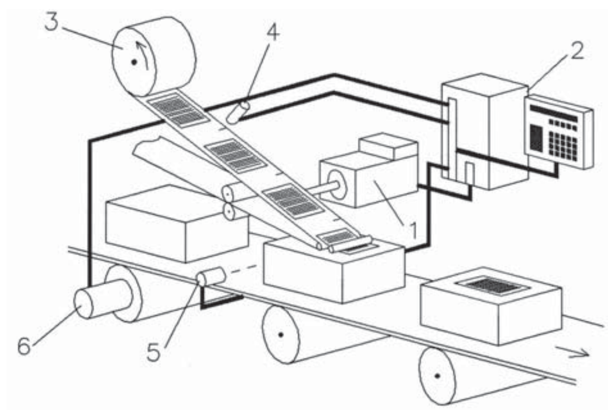

Fig. 1.15. Automatic labelling systems with a PM brushless motor: 1 — PM brushless motor servo drive, 2 — controller, 3 — spool of self-adhesive labels, 4 — registration mark sensor, 5 — box position sensor, 6 — conveyor speed encoder. Courtesy of *Parker Hannifin Corporation*, Rohnert Park, CA, U.S.A.

Small PM motors are especially demanded by manufacturers of computer hardware, automobiles, office equipment, medical equipment, instrumentation for measurements and control, robots, and handling systems. The 2002 world production of PM motors was estimated to be 4.68 billion units with a total value of U.S.$ 38.9 billion. Commutator motors account for 74.8% (3,500 million units), brushless motors account for 11.5% (540 million units) and stepping motors account for 13.7% (640 million units). From today's perspective, the Far East (Japan, China, and South Korea), America and Europe will remain the largest market area.

Advances in electronics and PM quality have outpaced similar improvements in associated mechanical transmission systems, making ball lead screws and gearing the limiting factors in motion control. For the small motor busi-

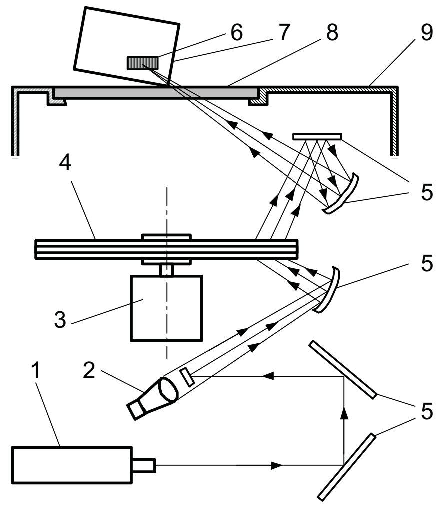

Fig. 1.16. Bar code scanner: 1 — laser, 2 — photodecoder converting laser beam into electric signal, 3 — PM brushless motor, 4 — holographic three-layer disk, 5 — mirror, 6 — bar code, 7 — scanned object, 8 — scan window, 9 — housing.

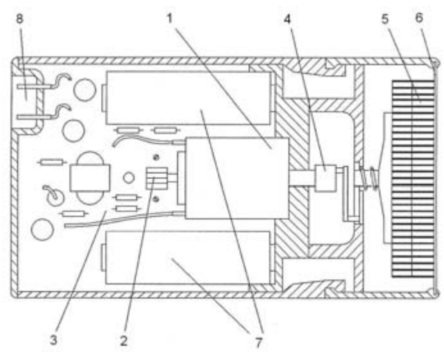

Fig. 1.17. Electric shaver with a PM brushless motor: 1 — PM brushless motor, 2 — position sensor, 3 — printed circuit board, 4 — cam-shaft, 5 — twizzer head, 6 — platinum-coated shaving foil, 7 — rechargeable battery, 8 — input terminals 110/220 V.

ness, a substantially higher integration of motor components will increasingly help to bridge this gap in the future [211]. However, there is always the question of cost analysis, which ultimately is the key factor for specific customer needs.

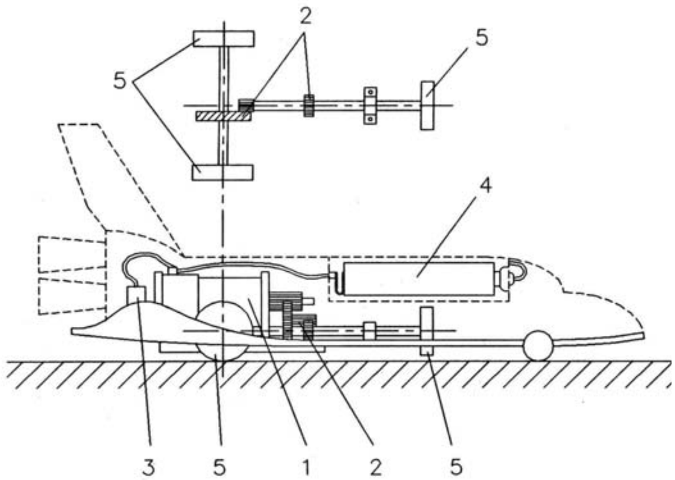

Fig. 1.18. Toy space shuttle: 1 — PM d.c. commutator motor, 2 — transmission, 3 — on–off switch, 4 — 1.5-V battery, 5 — driven wheels.

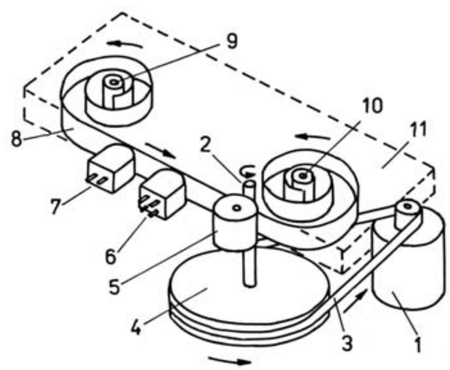

Fig. 1.19. Cassette deck: 1 — PM motor, 2 — capstan, 3 — belt, 4 — flywheel, 5 — pressure roller, 6 — rec/play head, 7 — erase head, 8 — tape, 9 — supply reel table, 10 — take-up reel table, 11 — cassette.

1.6 Applications of permanent magnet motors

PM motors are used in a broad power range from mWs to hundreds kWs. There are also attempts to apply PMs to large motors rated at minimum 1 MW. Thus, PM motors cover a wide variety of application fields, from stepping motors for wrist watches, through industrial drives for machine tools to large PM synchronous motors for ship propulsion (navy frigates, cruise ships, medium size cargo vessels and ice breakers) [109, 124, 278]. The application of PM electric motors includes:

- Industry (Figs 1.12, 1.14 1.13 and 1.15):
  - industrial drives, e.g., pumps, fans, blowers, compressors (Fig. 1.14), centrifuges, mills, hoists, handling systems, etc.
  - machine tools
  - servo drives
  - automation processes
  - internal transportation systems
  - robots
- Public life:
  - heating, ventilating and air conditioning (HVAC) systems
  - catering equipment
  - coin laundry machines
  - autobank machines
  - automatic vending machines
  - money changing machines
  - ticketing machines
  - bar code scanners at supermarkets (Fig. 1.16)
  - environmental control systems
  - clocks
  - amusement park equipment
- Domestic life (Fig. 1.17, 1.18 and 1.19):
  - kitchen equipment (refrigerators, microwave ovens, in-sink garbage disposers, dishwashers, mixers, grills, etc.)
  - bathroom equipment (shavers, hair dryers, tooth brushes, massage apparatus)
  - washing machines and clothes dryers
  - HVAC systems, humidifiers and dehumidifiers
  - vacuum cleaners
  - lawn mowers
  - pumps (wells, swimming pools, jacuzzi whirlpool tubs)
  - toys
  - vision and sound equipment
  - cameras
  - cellular phones
  - security systems (automatic garage doors, automatic gates)
- Information and office equipment (Figs. 1.20 and 1.21):
  - computers [161, 163]
  - printers
  - plotters
  - scanners
  - facsimile machines
  - photocopiers
  - audiovisual aids
- Automobiles with combustion engines (Fig. 1.22);

- Transportation (Figs. 1.23, 1.24, 1.25 and 1.27):
  - elevators and escalators
  - people movers
  - light railways and streetcars (trams)
  - electric road vehicles
  - aircraft flight control surface actuation
  - electric ships
  - electric boats
  - electric aircrafts (Fig. 1.27
- Defense forces (1.26):
  - tanks
  - missiles
  - radar systems
  - submarines
  - torpedos
- Aerospace:
  - rockets
  - space shuttles
  - satellites
- Medical and healthcare equipment:
  - dental handpieces (dentist's drills)
  - electric wheelchairs
  - air compressors
  - trotters
  - rehabilitation equipment
  - artificial heart motors
- Power tools (Fig. 1.28):
  - drills
  - hammers
  - screwdrivers
  - grinders
  - polishers
  - saws
  - sanders
  - sheep shearing handpieces [256]
- Renewable energy systems (Fig. 1.29)
- Research and exploration equipment (Fig. 1.30)

The automotive industry is the biggest user of PM d.c. commutator motors. The number of auxiliary d.c. PM commutator motors can vary from a few in an inexpensive car to about one hundred in a luxury car [175].

Small PM brushless motors are first of all used in computer hard disk drives (HDDs) and cooling fans. The 2002 worldwide production of computers is estimated to be 200 million units and production of HDDs approximately 250 million units.

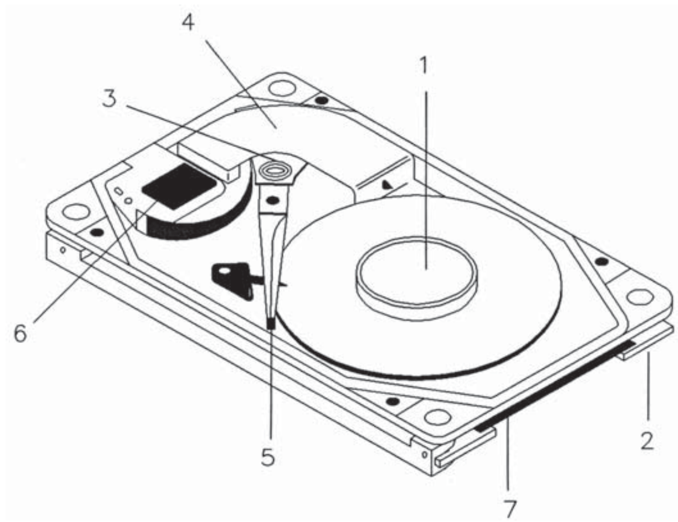

Fig. 1.20. Computer hard disk drive (HDD) with a PM brushless motor: 1 — in-hub PM brushless motor mounted in 2.5-inch disk, 2 — integrated interface/drive controller, 3 — balanced moving-coil rotary actuator, 4 — actuator PM, 5 — read/write heads, 6 — read/write preamplifier, 7 — 44-pin connector.

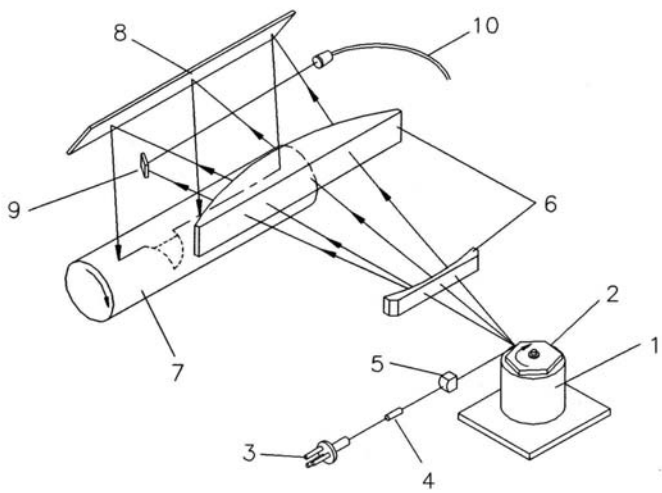

Fig. 1.21. Laser beam printer: 1 — stepping motor, 2 — scanner mirror, 3 — semiconductor laser, 4 — collimator lens, 5 — cylindrical lens, 6 — focusing lenses, 7 — photosensitive drum, 8 — mirror, 9 — beam detect mirror, 10 — optical fiber.

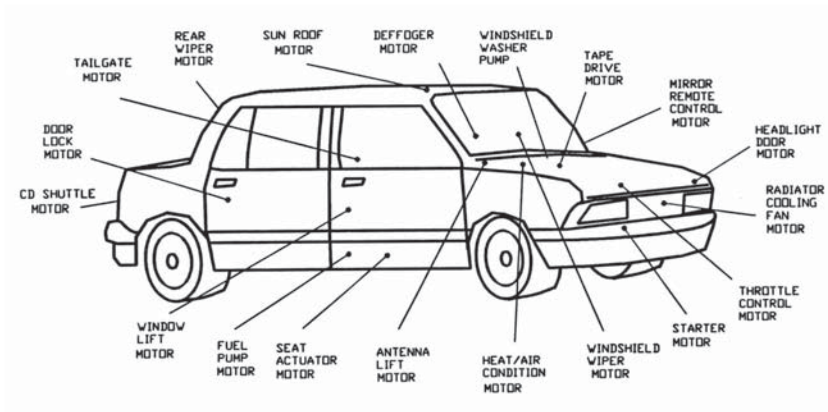

Fig. 1.22. PM motors installed in a car.

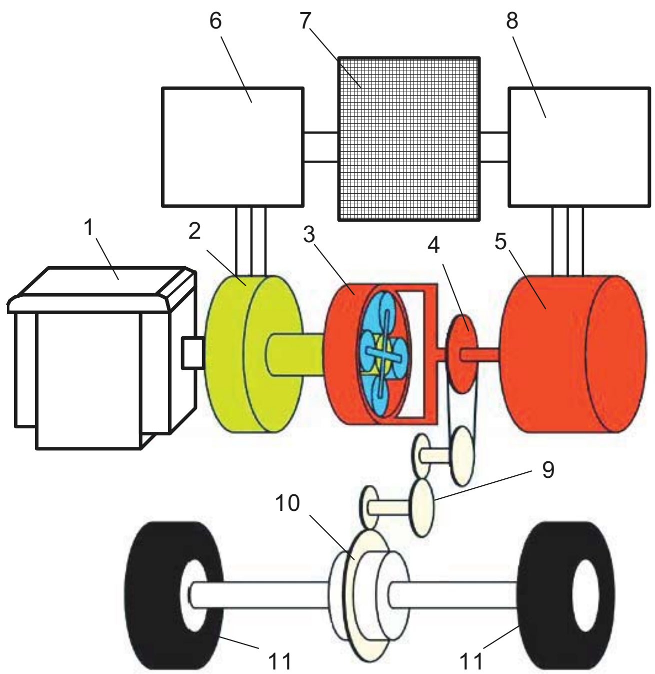

Fig. 1.23. Power train of Toyota Prius hybrid electric vehicle (HEV):1 — gasoline engine, 2 — PM brushless generator/starter (GS), 3 — power split device (PSD), 4 — silent chain, 5 — PM brushless motor/generator (MG), 6 — GS solid state converter, 7 — battery, 8 — MG solid state converter, 9 — reduction gears, 10 — differential, 11 — front wheels.

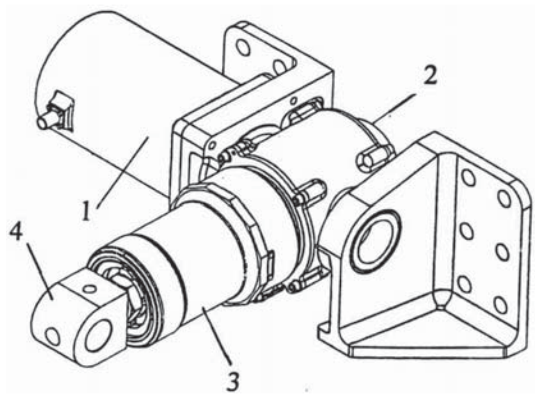

Fig. 1.24. Electromechanical actuator for flight control surfaces: 1 — brushless motor, 2 — gearbox, 3 — ball lead screw, 4 — clevis or spherical joint end.

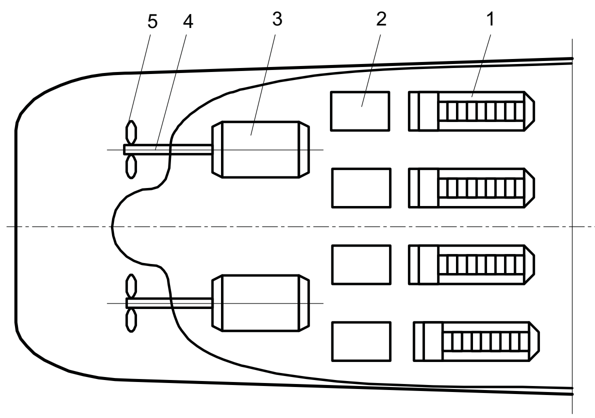

Fig. 1.25. Ship propulsion system with a PM brushless motor: 1 — diesel engine and synchronous generator, 2 — converter, 3 — large PM brushless motor, 4 — propeller shaft, 5 — propeller.

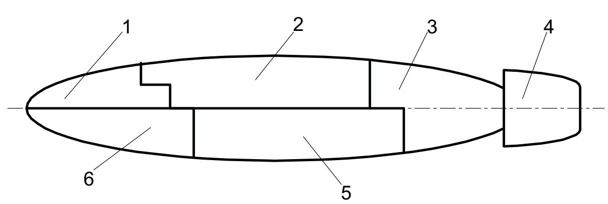

Fig. 1.26. Stealth torpedo: 1 — guiding system, 2 — conformal acoustic arrays, 3 — advanced rechargeable batteries, 4 — integrated PM brushless motor propulsor, 5 — active and passive noise control, 6 — synergic drag reduction.

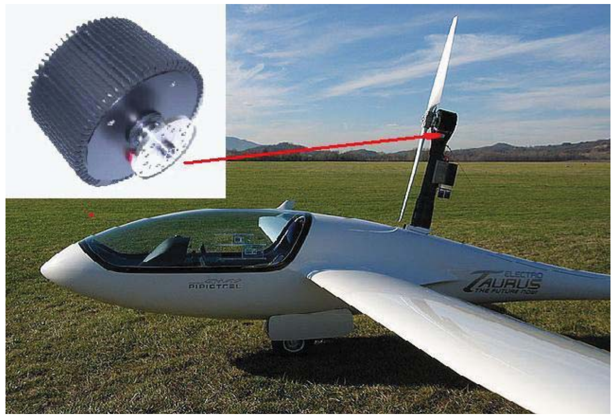

Fig. 1.27. *Taurus Electro* two-seat self launching glider with a 30 kW, 1800 rpm, 15.8 kg PM brushless motor. Photo courtesy of Pipistrel, Ajdovscina, Slovenia.

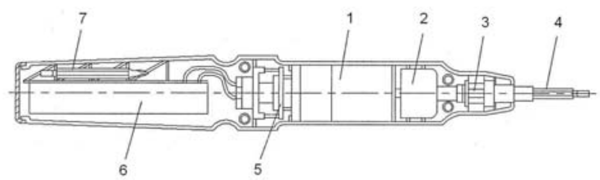

Fig. 1.28. Cordless electric screwdriver: 1 — PM d.c. commutator motor 3.6 V/240 mA, 2 — speed reducer, 3 — locker for manual screwing, 4 — screwdriver bit, 5 — forward-reverse rotation switch, 6 — rechargeable Ni-Cd battery, 7 — bit compartment.

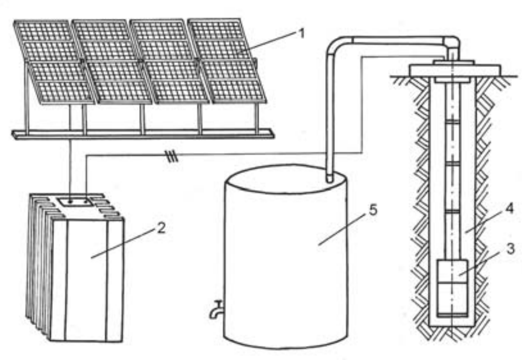

Fig. 1.29. Water pumping system for a remote population center: 1 — solar panels, 2 — inverter, 3 — submersible PM brushless motor–pump unit, 4 — well, 5 — water storage tank.

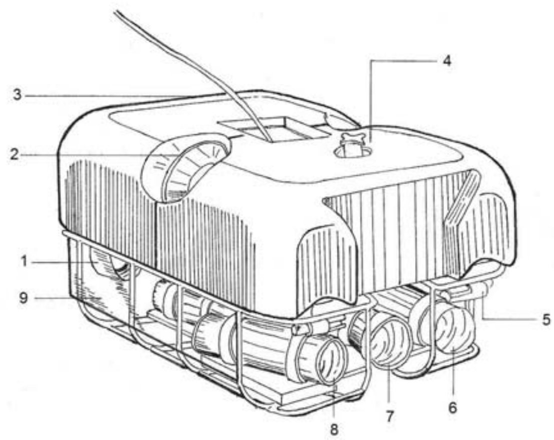

Fig. 1.30. Underwater robotic vehicle for remote inspection of hydro power plants: 1 — two thruster sets for forward and reverse (200-W d.c. brushless motors), 2 — two thruster sets for lateral and vertical (200-W d.c. brushless motors), 3 — buoyancy material, 4 — transponder, 5 — flood light, 6 — still camera, 7 — video camera, 8 — electric flash, 9 — cover. Courtesy of Mitsui Ocean Development and Eng. Co., Tokyo, Japan.

PM brushless motors rated from 50 to 100 kW seem to be the best propulsion motors for electric and hybrid road vehicles.

Given below are some typical applications of PM motors in industry, manufacturing processes, factory automation systems, domestic life, computers, transportation and clinical engineering:

- *Industrial robots* and *x, y-axis coordinate machines*: PM brushless motors
- *Indexing rotary tables*: PM stepping motors
- *X–Y tables*, e.g. for milling grooves across steel bars: PM brushless servo motors
- *Linear actuators with ball or roller screws*: PM brushless and stepping motors
- *Transfer machines for drilling a number of holes*: ball lead screw drives with PM brushless motors
- *Monofilament nylon winders*: PM d.c. commutator motor as a torque motor and PM brushless motor as a traverse motor (ball screw drive)
- *Mobile phones*: PM d.c. commutator or brushless vibration motors
- *Bathroom equipment*: PM commutator or brushless motors
- *Toys*: PM d.c. commutator motors
- *Computer hard disk drives* (HDD): PM brushless motors
- *Computer printers*: PM stepping motors
- *Cooling fans for computers and instruments*: PM brushless motors

- *Auxiliary motors for automobiles*: PM d.c. commutator and PM brushless motors
- *Gearless elevators*: PM brushless motors
- *Electric and hybrid electric vehicles* (EV and HEV): PM brushless motors of cylindrical or disk type
- *Ship propulsion*: large PM brushless motors or transverse flux motors (above 1 MW)
- *Submarine periscope drives*: direct-drive PM d.c. brushless torque motors
- *More electric aircraft* (MEE): PM brushless motors
- *Dental and surgical handpieces*: slotless PM brushless motors
- *Implantable blood pumps*: PM brushless motors integrated with impellers.

1.7 Mechatronics

A new technology called *mechatronics* emerged in the late 1970s. Mechatronics is the intelligent integration of mechanical engineering with microelectronics and computer control in product design and manufacture to give improved performance and cost saving. Applications of mechatronics can be found in the aerospace and defense industries, in intelligent machines such as industrial robots, automatic guided vehicles, computer-controlled manufacturing machines and in consumer products such as computer hard disk drives (HDD), video cassette players and recorders, cameras, CD players and quartz watches.

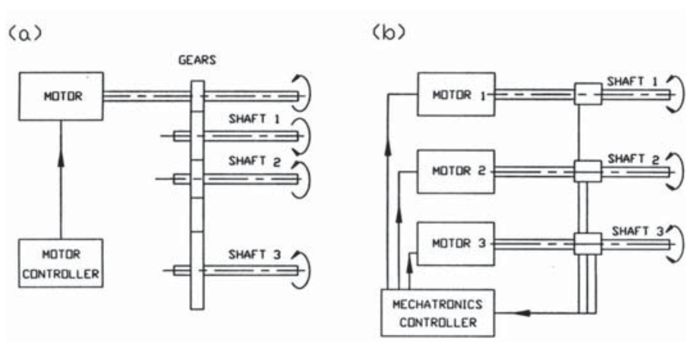

Fig. 1.31. Gear train design: (a) conventional system, (b) mechatronics system.

A typical example of a novel mechatronics application is in the control of multi-shaft motion. A gear train has traditionally been employed with the performance, i.e. speed, torque and direction of rotation determined by the motor and gear rated parameters as shown in Fig. 1.31a. Such a configuration is acceptable for constant speed of each shaft but where variable speeds are required, a different set of gears is needed for each gear ratio. In the mechatronics solution (Fig. 1.31b) each shaft is driven by an electronically controlled motor, e.g. a PM brushless motor with feedback which provides more flexibility than can be obtained from mechanical gear trains. By adding a microprocessor or microcomputer, any required motion of the mechanism can be programmed by software. The common term for this type of control system is *mechatronics control system* or *mechatronics controller*. The "electronic gearbox" is more flexible, versatile and reliable than the mechanical gearbox. It also reduces acoustic noise and does not require maintenance.

1.8 Fundamentals of mechanics of machines

1.8.1 Torque and power

The shaft torque T as a function of mechanical power P is expressed as

$$T = F\frac{D}{2} = \frac{P}{\Omega} = \frac{P}{2\pi n} \tag{1.1}$$

where $\Omega = 2\pi n$ is the angular speed and n is the rotational speed in rev/s.

Table 1.1. Basic formulae for linear and rotational motions

Linear motion			Rotational motion		
Quantity	Formula	Unit	Quantity	Formula	Unit
Linear displacement	$s = \theta r$	m	Angular displacement	θ	rad
Linear velocity	$v = ds/dt$ $v = \Omega r$	m/s	Angular velocity	$\Omega = d\theta/dt$	rad/s
Linear acceleration	$a = dv/dt$ $a_t = \alpha r$ $a_r = \Omega^2 r$	m/s^2	Angular acceleration	$\alpha = d\Omega/dt$	rad/s^2
Mass	m	kg	Moment of inertia	J	kgm^2
Force	$F = mdv/dt$ $= ma$	N	Torque	$T = Jd\Omega/dt$ $= J\alpha$	Nm
Friction force	Dds/dt $= Dv$	N	Friction torque	$Dd\theta/dt$ $= D\Omega$	Nm
Spring force	Ks	N	Spring torque	$K\theta$	Nm
Work	$dW = Fds$	Nm	Work	$dW = Td\theta$	Nm
Kinetic energy	$E_k = 0.5mv^2$	J or Nm	Kinetic energy	$E_k = 0.5J\Omega^2$	J
Power	$P = dW/dt$ $= Fv$	W	Power	P = dW/dt $= T\Omega$	W

1.8.2 Simple gear trains

In the simple trains shown in Fig. 1.32, let n_1, n_2 = speeds of 1 and 2, z_1 and z_2 = numbers of teeth on 1 and 2, D_1, D_2 = pitch circle diameters of 1 and 2.

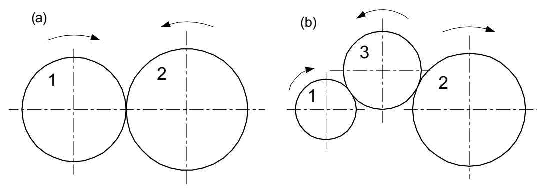

Fig. 1.32. Simple trains.

- in train according to Fig. 1.32a

$$\gamma = \frac{n_1}{n_2} = -\frac{z_2}{z_1} = -\frac{D_2}{D_1} \tag{1.2}$$

- in train according to Fig. 1.32b

$$\gamma = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{D_2}{D_1} \tag{1.3}$$

The negative sign signifies that 1 and 2 rotate in opposite directions. The idler, 3, Fig. 1.32b, does not affect the velocity ratio of 1 to 2 but decides on the directions of 2. The ratio $\gamma = z_2/z_1$ is called the *gear ratio*.

1.8.3 Efficiency of a gear train

Allowing for friction, the efficiency of a gear train is

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{P_2}{P_1} \tag{1.4}$$

Thus,

$$\eta = \frac{P_2}{P_1} = \frac{T_2(2\pi n_2)}{T_1(2\pi n_1)} = \frac{T_2 n_2}{T_1 n_1} \tag{1.5}$$

According to eqn (1.2) $n_2/n_1 = |z_1/z_2|$, so that eqn (1.5) becomes

$$\frac{T_2 n_2}{T_1 n_1} = \frac{T_2 z_1}{T_1 z_2}$$

The torque on wheel 1

$$T_1 = T_2 \frac{z_1}{z_2}\frac{1}{\eta} \tag{1.6}$$

1.8.4 Equivalent moment of inertia

In the simple trains shown in Fig. 1.32a, let J_1, J_2 = moments of inertia of rotating masses of 1 and 2, Ω_1 and Ω_2 = angular speed of 1 and 2, D_1, D_2 = pitch circle diameters of 1 and 2, $0.5J_1\Omega_1^2$, $0.5J_2\Omega_2^2$ = kinetic energy of 1 and 2, respectively.

The net energy supplied to a system in unit time is equal to the rate of change of its kinetic energy E_k (Table 1.1), i.e.

$$P = \frac{dE_k}{dt} = T\Omega_1$$

$$T\Omega_1 = \frac{d}{dt}[0.5J_1\Omega_1^2 + 0.5J_2\Omega_2^2] = 0.5\left(J_1 + \frac{\Omega_2^2}{\Omega_1^2}J_2\right) \times \frac{d}{dt}\Omega_1^2 \tag{1.7}$$

$$= 0.5\left(J_1 + \frac{\Omega_2^2}{\Omega_1^2}J_2\right) \times 2\Omega_1\frac{d\Omega_1}{dt} \tag{1.8}$$

The quantity $J_1 + (\Omega_2/\Omega_1)^2 J_2$ may be regarded as the equivalent moment of inertia of the gears referred to wheel 1. The moments of inertia of the various gears may be reduced to an equivalent moment of inertia of the motor shaft, i.e.

$$T = \left(J_1 + \frac{\Omega_2^2}{\Omega_1^2}J_2\right)\frac{d\Omega_1}{dt} = \left(J_1 + \frac{z_1^2}{z_2^2}J_2\right)\frac{d\Omega_1}{dt} \tag{1.9}$$

The equivalent moment of inertia is equal to the moment of inertia of each wheel in the train being multiplied by the square of its gear ratio relative to the reference wheel.

1.8.5 Rotor dynamics

All spinning shafts, even in the absence of external load, deflect during rotation. Fig. 1.33 shows a shaft with two rotating masses m_1 and m_2. The mass m_1 can represent a cylindrical rotor of an electric machine while the mass m_2 can represent a load. The mass of the shaft is m_{sh}. The combined mass of the rotor, load and shaft can cause deflection of the shaft that will create resonant vibration at a certain speed called *whirling* or *critical speed*. The frequency when the shaft reaches its critical speed can be found by calculating the frequency at which transverse vibration occurs. The critical speed in rev/s of the ith rotating mass can be found as [258]

$$n_{cri} = \frac{1}{2\pi}\sqrt{\frac{K_i}{m_i}} = \frac{1}{2\pi}\sqrt{\frac{g}{\sigma_i}} \tag{1.10}$$

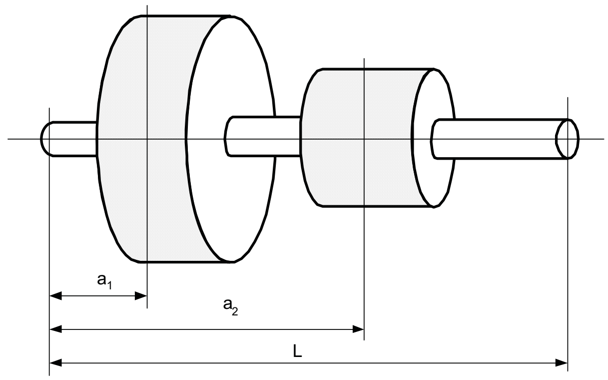

Fig. 1.33. Solid cylindrical shaft loaded with two masses m_1 and m_2.

where K_i is the stiffness of the ith rotor in N/m, m_i is the mass of the ith rotor in kg, $g = 9.81$ m/s^2 is the acceleration due to gravity and δ_i is the static deflection at the ith position of the rotor due to the ith rotor only, i.e.,

$$\sigma_i = \frac{m_i g a_i^2 (L - a_i)^2}{3 E_i I_i L} \tag{1.11}$$

In the above equation E_i is the modulus of elasticity (for steel $E = 200 \times 10^9$ Pa), I_i is the area moment of inertia of cross-sectional area, L is the length of the shaft and a_i is the location of the ith rotor from the left end of the shaft (Fig. 1.33). The area moment of inertia can be found as

$$I_i = \frac{\pi D_i^4}{64} \tag{1.12}$$

The resultant angular critical speed $\Omega_{cr} = 2\pi n_{cr}$ for the shaft loaded with i number of rotors ($\Omega_{cri} = 2\pi n_{cri}$) can be found on the basis of Dunkerley's equation [88] as

$$\frac{1}{\Omega_{cr}^2} = \sum_i \frac{1}{\Omega_{cri}^2} \tag{1.13}$$

or Rayleigh's equation [258]

$$\Omega_{cr} = \sqrt{\frac{g \sum_i (m_i \sigma_i)}{\sum_i (m_i \sigma_i^2)}} \tag{1.14}$$

The shaft is also considered a rotor with mass m_{sh} concentrated at $0.5L$ where L is the length of the shaft (bearing–to–bearing).

Dunkerley's empirical method uses the frequencies that each individual load creates when each load acts alone and then combines them to give an approximation for the whole system [88]. Thus eqn (1.13) is an approximation to the *first natural frequency* of vibration of the system which is assumed nearly equal to the critical speed of rotation. Rayleigh's method is based on the fact that the maximum kinetic energy must be equal to maximum potential energy for a conservative system under free vibration [258].

1.8.6 Mechanical characteristics of machines

In general, the mechanical characteristic $T = f(\Omega)$ of a machine driven by an electric motor can be described by the following equation:

$$T = T_r \left(\frac{\Omega}{\Omega_r} \right)^{\beta} \tag{1.15}$$

where T_r is the resisting torque of the machine at rated angular speed Ω_r, $\beta = 0$ for hoists, belt conveyors, rotating machines and vehicles (constant torque machines), $\beta = 1$ for mills, callanders, paper machines and textile machines, $\beta = 2$ for rotary pumps, fans, turbocompressors and blowers.

1.9 Torque balance equation

An electromechanical system can simply be described with the aid of the following torque balance equation,

$$J \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt} + K\theta = \pm T_d \mp T_{sh} \tag{1.16}$$

where J is the *moment of inertia* of the system in kgm^2 assumed as constant ($dJ/dt = 0$), D is the *damping coefficient* in Nm s/rad, K is the *stiffness coefficient* or *spring constant* in Nm/rad, T_d is the instantaneous electromagnetic torque developed by the motor, T_{sh} is the instantaneous external (shaft) passive load torque, θ is the rotor angular displacement, $T_d > T_{sh}$ for acceleration, $T_d < T_{sh}$ for deceleration. Assuming $D = 0$ and $K = 0$ the torque balance equation (1.16) becomes

$$J \frac{d^2\theta}{dt^2} \approx \pm T_d \mp T_{sh} \tag{1.17}$$

1.10 Evaluation of cost of a permanent magnet motor

The cost of an electrical machine is a function of a large number of variables. The cost can be evaluated only approximately because it depends on:

- number of electrical machines of the same type manufactured per year
- manufacturing equipment (how modern is the equipment, level of automation, production capacity per year, necessary investment, etc.)

- organization of production process (engineering staff–to–administrative and supporting staff ratio, qualification and experience of technical management, overhead costs, productivity of employees, small company or large corporation, company culture, etc.)
- cost of labor (low in third world countries, high in North America, Europe and Japan)
- quality of materials (good quality materials cost more) and many other aspects

It is impossible to take into account all these factors in a general mathematical model of costs. A logical approach is to select the most important components of the total cost and express them as functions of dimensions of the machine [186].

The most important costs of an electrical machine can be expressed by the following approximate equation:

$$C = k_N(C_w + C_c + C_{PM} + C_{sh} + C_0) \tag{1.18}$$

where $k_N \leq 1$ is the coefficient depending on the number of manufactured machines per annum, C_w is the cost of winding, C_c is the cost of ferromagnetic core and components dependent on the size of core (frame, end disks, bearings, etc), C_{PM} is the cost of PMs, C_{sh} is the cost of shaft and C_0 is the cost of all other components independent of the shape of the machine, e.g. nameplate, encoder, terminal board, terminal leads, commutator in d.c. brush machine, etc.

The cost of winding is [186]

$$C_w = k_{sp}k_{ii}k_{sr}\rho_{Cu}c_{Cu}V_{sp} \tag{1.19}$$

where $k_{sp} < 1$ is the slot space (fill) factor, $k_{ii} > 1$ is the coefficient of the cost of fabrication of coils including placing in slots, insulation, impregnation, etc., $k_{sr} \geq 1$ is the cost of the stator and rotor winding–to–the cost of the stator winding ratio (if the rotor winding exists, e.g., damper), ρ_{Cu} is the specific mass density of the conductor material (copper) in kg/m^3, c_{Cu} is the cost of conductor per kilogram and V_{sp} is the space designed for the winding and insulation in m^3.

The cost of a ferromagnetic core consists of the cost of laminated parts C_{cl} and other material parts, e.g., sintered powder parts C_{csp}, i.e.,

$$C_c = k_p(C_{cl} + C_{csp}) \tag{1.20}$$

where $k_p > 1$ is the coefficient accounting for the cost $\sum C_{ci}$ of all machine parts dependent on the dimensions of the stator core (frame, end plates, bearings, etc.) expressed as

$$k_p = 1 + \frac{\sum C_{ci}}{C_c} \tag{1.21}$$

The cost of laminated core

$$C_{cl} = k_u k_i k_{ss} \rho_{Fe} c_{Fe} \frac{\pi D_{out}^2}{4} \sum L \tag{1.22}$$

where $k_u > 1$ is the coefficient of utilization of electrotechnical steel sheet or strip (total surface of sheet/strip corresponding to single lamination to the surface of the lamination), $k_i < 1$ is the stacking (insulation) factor, $k_{ss} > 1$ is the coefficient accounting for the cost of stamping, stacking and other operations, ρ_{Fe} is the specific mass density of electrotechnical steel, c_{Fe} is the cost of electrotechnical sheet steel per kilogram, D_{out} is the outer diameter of the core and $\sum L$ is the total length of laminated stacks (the stack can be divided into segments).

The cost of sintered powder core

$$C_{csp} = k_{sh} \rho_{sp} c_{sp} V_{sp} \tag{1.23}$$

where $k_{sh} > 1$ is the coefficient accounting for the increase of the cost of sintered powder part dependent on the complexity of its shape (related to a simple shape, e.g., a cube), ρ_{sp} is the specific mass density of the sintered powder material, c_{sp} is the cost of sintered powder material per kilogram, V_{sp} is the volume of the sintered powder part. The cost of the solid steel core can be calculated in a similar way.

The cost of PMs is

$$C_{PM} = k_{shPM} k_{magn} \rho_{PM} c_{PM} V_M \tag{1.24}$$

where $k_{shPM} > 1$ is the coefficient accounting for the increase of the cost of PMs due complexity of their shape (related to a simple shape, e.g., a cube), $k_{magn} > 1$ is the coefficient taking into account the cost of magnetization of PMs, ρ_{PM} is the specific mass density of the PM material, c_{PM} is the cost of PM material per kilogram and V_M is the volume of PMs.

The cost of the shaft

$$C_{sh} = k_{ush} k_m \rho_{steel} c_{steel} V_{sh} \tag{1.25}$$

where $k_{ush} > 1$ is the coefficient of utilization of the round steel bar (total volume of the steel bar to the volume of the shaft), $k_m > 1$ is the coefficient accounting for the cost of machining, ρ_{steel} is the specific mass density of steel, c_{steel} is the cost of steel bar per kilogram, V_{sh} is the shaft volume.

Numerical examples

Numerical example 1.1

Find the steady-state torque, output power, and shaft moment of inertia of an electric motor propelling a rolling mill as in Fig. 1.34. The speed of the

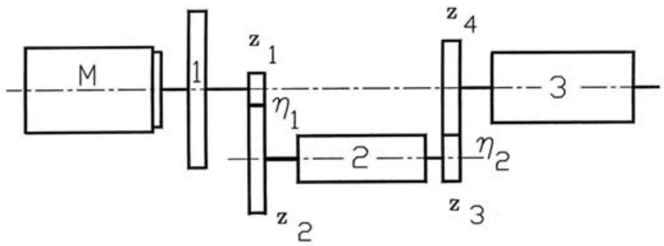

Fig. 1.34. Electric motor driven rolling mill. *Numerical example 1.1*.

motor is $n = 730$ rpm. The flywheel and rollers are made of steel with the specific mass density $\rho = 7800$ kg/m^3.

Solid steel flywheel 1: diameter $D_1 = 1.5$ m, thickness $l_1 = 0.2$ m.

First roller 2: diameter $D_2 = 0.4$ m, length $l_2 = 0.8$ m, circumferential force $F_2 = 20$ kN, number of teeth of the first gear $z_1 = 15$, $z_2 = 35$, efficiency of the first gear $\eta_1 = 0.87$.

Second roller 3: diameter $D_3 = 0.5$ m, length $l_3 = 1.2$ m, circumferential force $F_3 = 14$ kN, number of teeth of the second gear $z_3 = 20$, $z_4 = 45$, efficiency of the second gear $\eta_2 = 0.9$.

Solution

The shaft (load) torque according to eqn (1.6)

$$T_{sh} = F_2 \frac{D_2}{2} \frac{z_1}{z_2} \frac{1}{\eta_1} + F_3 \frac{D_3}{2} \frac{z_1}{z_2} \frac{z_3}{z_4} \frac{1}{\eta_1} \frac{1}{\eta_2} = 2.82 \text{ kNm}$$

where the gear ratio is $\gamma = z_2/z_1$.

Output power of the motor

$$P_{out} = 2\pi n T_{sh} = 2\pi \times (730/60) \times 2820 = 216 \text{ kW}$$

The mass of flywheel

$$m_1 = \rho \frac{\pi D_1^2}{4} l_1 = 2757 \text{ kg}$$

The mass of the first roller

$$m_2 = \rho \frac{\pi D_2^2}{4} l_2 = 785 \text{ kg}$$

The mass of the second roller

$$m_3 = \rho \frac{\pi D_3^2}{4} l_3 = 1840 \text{ kg}$$

Moment of inertia of the flywheel

$$J_1 = m_1 \frac{D_1^2}{8} = 776 \text{ kgm}^2$$

Moment of inertia of the first roller

$$J_2 = m_2 \frac{D_2^2}{8} = 15.7 \text{ kgm}^2$$

Moment of inertia of the second roller

$$J_3 = m_3 \frac{D_3^2}{8} = 57.5 \text{ kgm}^2$$

The total moment of inertia of the system with respect to the motor shaft according to eqn (1.9)

$$J = J_1 + J_2 \left(\frac{z_1}{z_2}\right)^2 + J_3 \left(\frac{z_1 z_3}{z_2 z_4}\right)^2 = 781 \text{ kgm}^2$$

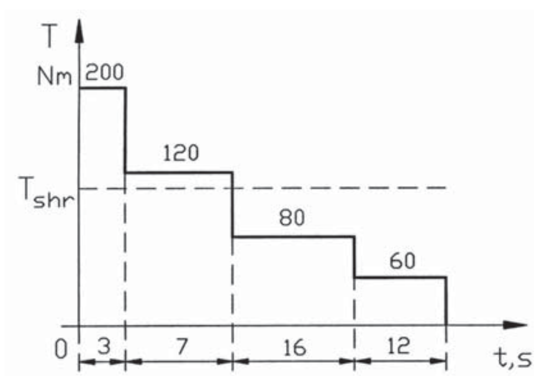

Fig. 1.35. Torque profile of the electric motor. *Numerical example 1.2*.

Numerical example 1.2

A 12-kW, 1000-rpm electric motor operates with almost constant speed according to the torque profile given in Fig. 1.35. The overload capacity factor $k_{ocf} = T_{max}/T_{shr} = 2$. Find the thermal utilization coefficient of the motor.

Solution

The required rated shaft torque

$$T_{shr} = \frac{P_{out}}{2\pi n} = \frac{12,000}{2\pi \times (1000/60)} = 114.6 \text{ Nm}$$

The *rms* torque based on the given duty cycle

$$T_{rms}^2(t_1+t_2+\ldots+t_n) = T_1^2 t_1+T_2^2 t_2+\ldots+T_n^2 t_n \quad \text{or} \quad T_{rms}^2 \sum t_i = \sum T_i^2 t_i$$

Thus

$$T_{rms} = \sqrt{\frac{\sum T_i^2 t_i}{\sum t_i}} = \sqrt{\frac{200^2 \times 3 + 120^2 \times 7 + 80^2 \times 16 + 60^2 \times 12}{3 + 7 + 16 + 12}}$$

$$= 95.5 \text{ Nm}$$

Note that in electric circuits the *rms* or effective current is

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

since the average power delivered to the resistor is $P = RI_{rms}^2$.

The maximum torque in Fig. 1.35 cannot exceed the rated shaft torque times overload capacity factor $k_{ocf} \times T_{shr}$. Also, the required T_{shr} should be greater than or equal to T_{rms}.

The coefficient of thermal utilization of the motor

$$\frac{T_{rms}}{T_{sh}} \times 100\% = \frac{95.5}{114.6} \times 100\% = 83.3\%$$

Numerical example 1.3

The required torque and speed profiles of a servo drive are given in Fig. 1.36. At constant speed 2500 rpm the load shaft torque is $T_{sh} = 1.5$ Nm. The load intertia subjected to the motor axis is $J_L = 0.004$ kgm^2. Assuming the servomotor intertia $J_M = 0.5J_L$, select a PM brushless servomotor.

Solution

The mechanical balance according to eqn (1.17) is expressed as

$$2\pi J \frac{\Delta n}{\Delta t} = T_d \pm T_{sh}$$

where T_d is the electromagnetic torque developed by the motor for acceleration or braking, the "$-$" sign is for acceleleration, and the "+" sign is for deceleration. The motor torque required for acceleration

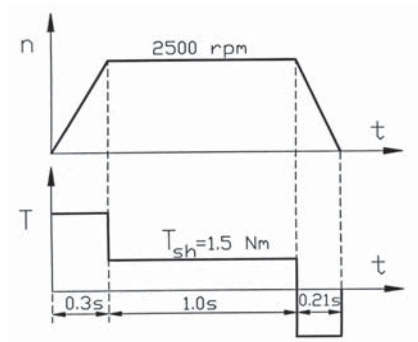

Fig. 1.36. Speed and torque profiles of a servo drive. *Numerical example 1.3*.

$$T_d = 2\pi(J_M + J_L)\frac{\Delta n}{\Delta t} + T_{sh} = \frac{2\pi}{60}(0.004 + 0.002)\frac{2500 - 0}{0.3 - 0} + 1.5 \approx 6.74 \text{ Nm}$$

The motor torque required for braking

$$T_d = 2\pi(J_M + J_L)\frac{\Delta n}{\Delta t} - T_{sh} = \frac{2\pi}{60}(0.004 + 0.002)\frac{2500}{0.21} - 1.5 \approx 5.98 \text{ Nm}$$

The *rms* torque

$$T_{rms} = \sqrt{\frac{6.74^2 \times 0.3 + 1.5^2 \times 1.0 + 5.98^2 \times 0.21}{0.3 + 1.0 + 0.21}} = 3.93 \text{ Nm}$$

The output power calculated for the *rms* torque

$$P_{out} = T_{rms}(2\pi n) = 3.93 \times 2\pi \times \frac{2500}{60} = 1030 \text{ W}$$

The overload capacity factor for $T_{dmax} = 6.74$ Nm

$$\frac{T_{dmax}}{T_{rms}} = \frac{6.74}{3.93} = 1.715$$

A PM brushless motor rated at 1.1 kW with minimum 1.8 overload capacity factor is recommended.

Numerical example 1.4

A 10 kW, 1450 rpm electric motor has been used to drive the following machines: (a) a hoist ($\beta = 0$), (b) a mill ($\beta = 1$) and (c) a fan ($\beta = 2$). The load torque in each case is 60 Nm. Find the drop in mechanical power if the speed is reduced to $n = 1200$ rpm.

Solution

The output power delivered by the motor at the resisting torque $T_r = 60$ Nm and $n_r = 1450$ rpm

$$P_{outr} = T_r \Omega_r = T_r(2\pi n_r) = 60(2\pi \frac{1450}{60}) = 9111 \text{ kW}$$

As the speed is reduced to $n = 1200$ rpm, the load torque is subject to a change according to eqn (1.15), i.e.,

(a) for the hoist

$$T = 60 \left(\frac{1200}{1450}\right)^0 = 60 \text{ Nm}$$

$$P_{out} = T(2\pi n) = 60 \times \left(2\pi \frac{1200}{60}\right) = 7540 \text{ W}$$

(b) for the mill

$$T = 60 \left(\frac{1200}{1450}\right)^1 = 49.7 \text{ Nm}$$

$$P_{out} = T(2\pi n) = 49.7 \times \left(2\pi \frac{1200}{60}\right) = 6245 \text{ W}$$

(c) for the fan

$$T = 60 \left(\frac{1200}{1450}\right)^2 = 41.1 \text{ Nm}$$

$$P_{out} = T(2\pi n) = 41.1 \times \left(2\pi \frac{1200}{60}\right) = 5165 \text{ W}$$

The mechanical power at reduced speed and referred to the rated power is

(a) for the hoist

$$\frac{7540}{9111} \times 10\% = 82.7\%$$

(b) for the mill

$$\frac{6245}{9111} \times 10\% = 68.5\%$$

(c) for the fan

$$\frac{5165}{9111} \times 10\% = 56.7\%$$

Numerical example 1.5

A 25-kg disk is pin-supported at its center. The radius of the disk is $r = 0.2$ m. It is acted upon by a constant force F = 20 N which is applied to a cord wrapped around its periphery. Determine the number of revolutions it must make to attain an angular velocity of 25 rad/s starting from rest. Neglect the mass of the cord.

Solution

According to Table 1.1 the kinetic energy $E_k = 0.5J\Omega^2$ where the moment of inertia is $J = 0.5mr^2$. Initially, the disk is at rest ($\Omega_1 = 0$), so that

$$E_{k1} = 0$$

The kinetic energy at $\Omega_2 = 25$ rad/s

$$E_{k2} = \frac{1}{2}J\Omega_2^2 = \frac{1}{4}mr^2\Omega_2^2 = \frac{1}{4}25 \times 0.2^2 \times 25^2 = 156.25 \text{ J}$$

The constant force F does positive work $W = Fs$ as the cord moves downward where $s = \theta r$ (Table 1.1). Thus, the *principle of work and energy* may be written as

$$E_{k1} + Fs = E_{k2} \qquad \text{or} \qquad E_{k1} + Fr\theta = E_{k2}$$

Therefore

$$\theta = \frac{E_{k2} - E_{k1}}{Fr} = \frac{156.25 - 0}{20 \times 0.2} = 39.06 \text{ rad}$$

The number of revolutions is

$$39.06 \text{ rad} \left(\frac{1\text{rev}}{2\pi \text{ rad}}\right) = 6.22 \text{ rad}$$

Numerical Example 1.6

Find the critical speed of rotation of the system consisting of a cylindrical rotor (laminated stack with PMs), steel shaft and driven wheel. Moduli of elasticity, specific mass densities, diameters and widths (lengths) are as follows:

(a) $E_1 = 200 \times 10^9$ Pa, $\rho_1 = 7600$ kg/m^3, $D_1 = 0.24$ m, $w_1 = 0.24$ m for the rotor;
(b) $E_2 = 200 \times 10^9$ Pa, $\rho_2 = 7650$ kg/m^3, $D_2 = 0.4$, $w_2 = 0.15$ m for the driven wheel;
(c) $E_{sh} = 210 \times 10^9$ Pa, $\rho_{sh} = 7700$ kg/m^3, $D_{sh} = 0.0508$ m, $L = 0.76$ m for the shaft.

The location of the rotor from the left end of the shaft is $a_1 = 0.28$ m and the location of the driven wheel from the same end of the shaft is $a_2 = 0.6$ m (Fig. 1.33). The acceleration of gravity is 9.81 m/s^2.

Solution

Mass of rotor

$$m_1 = \rho_1 \frac{\pi D_1^2}{4} w_1 = 7600 \frac{\pi 0.24^2}{4} \times 0.24 = 82.52 \text{ kg}$$

Mass of driven wheel

$$m_2 = \rho_2 \frac{\pi D_2^2}{4} w_2 = 7650 \frac{\pi 0.4^2}{4} \times 0.15 = 144.2 \text{ kg}$$

Mass of shaft

$$m_{sh} = \rho_{sh} \frac{\pi D_{sh}^2}{4} L = 7700 \frac{\pi 0.0508^2}{4} \times 0.76 = 11.86 \text{ kg}$$

Area moment of inertia of the rotor according to eqn (1.12)

$$I_1 = \frac{\pi 0.24^4}{64} = 1.629 \times 10^{-4} \text{ m}^4$$

Area moment of inertia of the driven wheel to eqn (1.12)

$$I_2 = \frac{\pi 0.4^4}{64} = 12.57 \times 10^{-4} \text{ m}^4$$

Area moment of inertia of the shaft according to eqn (1.12)

$$I_{sh} = \frac{\pi 0.0508^4}{64} = 3.269 \times 10^{-7} \text{ m}^4$$

Static deflection of the shaft at position of rotor due to rotor only as given by eqn (1.11)

$$\sigma_1 = \frac{82.52 \times 9.81 \times 0.28^2 (0.76 - 0.28)^2}{3 \times 200 \times 10^9 \times 1.629 \times 10^{-4} \times 0.76} = 1.969 \times 10^{-7} \text{ m}$$

Static deflection of the shaft at position of driven wheel due to driven wheel only as given by eqn (1.11)

$$\sigma_2 = \frac{144.2 \times 9.81 \times 0.6^2 (0.76 - 0.6)^2}{3 \times 200 \times 10^9 \times 12.57 \times 10^{-4} \times 0.76} = 2.275 \times 10^{-8} \text{ m}$$

Static deflection of shaft due to shaft only as given by eqn (1.11)

$$\sigma_{sh} = \frac{11.86 \times 9.81 \times 0.38^2 (0.76 - 0.38)^2}{3 \times 210 \times 10^9 \times 3.269 \times 10^{-7} \times 0.76} = 1.55 \times 10^{-5} \text{ m}$$

where the midpoint of the shaft is $0.5L = 0.5 \times 0.76 = 0.38$ m. Thus, critical speeds according to eqn (1.10) are

- Critical speed of the rotor

$$n_{cr1} = \frac{1}{2\pi}\sqrt{\frac{9.81}{1.969 \times 10^{-7}}} = 1123.4 \text{ rev/s} = 67\ 405.2 \text{ rpm}$$

- Critical speed of the driven wheel

$$n_{cr2} = \frac{1}{2\pi}\sqrt{\frac{9.81}{2.275 \times 10^{-8}}} = 3304.9 \text{ rev/s } = 198\ 292.5 \text{ rpm}$$

- Critical speed of the shaft

$$n_{crsh} = \frac{1}{2\pi}\sqrt{\frac{9.81}{1.55 \times 10^{-5}}} = 126.6 \text{ rev/s } = 7596.8 \text{ rpm}$$

Critical angular speeds for the rotor is $\Omega_{cr1} = 2\pi 1123.4 = 7058.7$ rad/s, for the driven wheel is $\Omega_{cr2} = 2\pi 3304.9 = 20\ 765.1$ rad/s and for the shaft is $\Omega_{crsh} = 2\pi 126.6 = 795.5$ rad/s. According to Dunkerley equation

$$x = \frac{1}{\Omega_{cr1}^2} + \frac{1}{\Omega_{cr2}^2} + \frac{1}{\Omega_{crsh}^2} = \frac{1}{7058.7^2} + \frac{1}{20\ 765.1^2} + \frac{1}{795.5^2} = 1.602 \times 10^{-6} \text{ s}^2/\text{rad}^2$$

Critical angular speed of the system as given by eqn (1.13)

$$\Omega_{cr} = \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{1.602 \times 10^{-6}}} = 790 \text{ rad/s}$$

Critical speed of rotation of the system according to Dunkerley equation

$$n_{cr} = \frac{\Omega_{cr}}{2\pi} = \frac{790}{2\pi} = 125.7 \text{ rev/s } = 7543.6 \text{ rpm}$$

Critical speed of rotation of the system according to Rayleigh's method - eqn (1.14)

$$n_{cr} = \frac{1}{2\pi}\sqrt{\frac{9.81(82.52 \times 1.969 \times 10^{-7} + 144.2 \times 2.275 \times 10^{-8} + 11.86 \times 1.55 \times 10^{-5})}{82.52 \times (1.969 \times 10^{-7})^2 + 144.2 \times (2.275 \times 10^{-8})^2 + 11.86 \times (1.55 \times 10^{-5})^2}}$$
$$= 133.1 \text{ rev/s } = 7985.5 \text{ rpm}$$

The results according to Dunkerley [88] and Rayleigh [258] are not the same.

Numerical example 1.7

Estimate the cost of a 3-phase, 7.5 kW PM brushless servo motor with sintered NdFeB PMs and laminated stator and rotor cores. The mass of stator copper conductors is $m_{Cu} = 7.8$ kg, mass of stack $m_{Fe} = 28.5$ kg (stator and rotor), mass of PMs $m_{PM} = 2.10$ kg and mass of shaft $m_{sh} = 6.2$ kg. The cost of materials in U.S. dollars per kilogram is: copper conductor $c_{Cu} = 5.55$, steel laminations $c_{Fe} = 2.75$, NdFeB magnets $c_{PM} = 54.50$ and shaft steel $c_{steel} = 0.65$. The cost of components independent of the machine geometry (nameplate, encoder, terminal leads, terminal board) is $C_0 = \$146.72$.

Coefficients taking into account manufacturing, utilization, complexity and economic factors of PM brushless motors are as follows:

- coefficient dependent of the number of machines manufactured per annum, $k_N = 0.85$ (10,000 machines manufactured per year)
- coefficient taking into account the cost of frame, end bells and bearings, $k_p = 1.62$
- coefficient of the cost of fabrication of coils (insulation, assembly, impregnation), $k_{ii} = 2.00$
- coefficient taking into account the cost of the rotor winding, $k_{sr} = 1.0$ (no rotor winding)
- coefficient of utilization of electrotechnical steel, $k_u = 1.3$
- stacking (insulation) factor, $k_i = 0.96$
- coefficient including the cost of stamping, stacking and other operations, $k_{ss} = 1.4$
- coefficient accounting for the increase of the cost of PMs due to complexity of their shape, $k_{shPM} = 1.15$
- coefficient including the cost of magnetization of PMs, $k_{magn} = 1.1$
- total volume of the steel bar to the volume of the shaft, $k_{ush} = 1.94$
- coefficient taking into account the cost of machining of the shaft, $k_m = 3.15$

It has been assumed for cost analysis that 10,000 machines are manufactured per year.

Solution

The cost of the laminated stack with frame (enclosure), end bells and bearings

$$C_{cl} = k_p k_u k_i k_{ss} m_{Fe} c_{Fe} = 1.62 \times 1.3 \times 0.96 \times 1.4 \times 28.5 \times 2.75) = \$221.84$$

The cost of the copper winding

$$C_w = k_{ii} k_{sr} m_{Cu} c_{Cu} = 2.00 \times 1.0 \times 7.8 \times 5.55 = \$86.58$$

The cost of PMs

$$C_{PM} = k_{shPM} k_{magn} m_{PM} c_{PM} = 1.15 \times 1.1 \times 2.10 \times 54.50 = \$144.78$$

The cost of the shaft

$$C_{sh} = k_{ush} k_m m_{sh} c_{steel} = 1.94 \times 3.15 \times 6.2 \times 0.65 = \$24.63$$

Total cost of the motor

$$C = k_N (C_{cl} + C_w + C_{PM} + C_{sh} + C_0)$$

$$= 0.85(221.84 + 86.58 + 144.78 + 24.63 + 146.72) = \$530.87$$

The evaluated cost of 10,000 machines (annual production) is $10,000 \times 530.87 = \$5,308,700$.